Indeterminate Form — Class - B.Sc I (Hons)

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Main objective of Indeterminate form :→

$$\lim_{x\to a} \frac{\varphi(x)}{\psi(x)}$$ calculate $\varphi(a)$ and $\psi(a)$

if $\varphi(a) \neq 0$, $\psi(a) \neq 0$ then $\lim_{x\to a} \frac{\varphi(x)}{\psi(x)} = \frac{\varphi(a)}{\psi(a)}$

If $\varphi(a) = 0$, $\psi(a) \neq 0$ then $\lim_{x\to a} \frac{\varphi(x)}{\psi(x)} = \frac{0}{\psi(a)} = 0$

If $\varphi(a) \neq 0$, $\psi(a) = 0$ then $\lim_{x\to a} \frac{\varphi(x)}{\psi(x)} = \frac{\varphi(a)}{0} = \infty$

But if $\varphi(a) = 0$, $\varphi(a) = 0$ In this case the rule becomes inapplicable for the quotient Takes the form $\frac{0}{0}$ which is Vague and Misleading

The form $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\times\infty$, $\infty-\infty$, 0^0, ∞^0, 1^∞ which are meaningless are Termed as Indeterminate form

Working Rule for different Form.

1. Form $\frac{0}{0}$:→

$$\lim_{x\to a} \frac{\varphi(x)}{\psi(x)} = \lim_{x\to a} \frac{\varphi'(x)}{\psi'(x)} = \frac{\varphi'(a)}{\psi'(a)}$$

Ex :→ $\lim\limits_{x \to 0} \frac{a^x - 1}{x} \quad \left[\frac{0}{0}\right]$

$\Rightarrow \lim\limits_{x \to 0} \frac{a^x \log a}{1} = a^0 \log a = \log a$

Ex:→ $\lim\limits_{x \to 0} \frac{a^x - b^x}{x} \quad \left[\frac{0}{0}\right]$

$\Rightarrow \lim\limits_{x \to 0} \frac{\frac{d}{dx}(a^x - b^x)}{\frac{d}{dx}(x)}$

$= \lim\limits_{x \to 0} \frac{a^x \log a - b^x \log b}{1}$

$= \log a - \log b.$

$= \log \frac{a}{b}.$

Ex:→ $\lim\limits_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2} \quad \left[\frac{0}{0}\right]$

$= \lim\limits_{x \to 0} \frac{\frac{d}{dx}(e^x + e^{-x} - 2)}{\frac{d}{dx}(x^2)}$

$= \lim\limits_{x \to 0} \frac{e^x - e^{-x}}{2x} \quad \left[\frac{0}{0}\right]$

$= \lim\limits_{x \to 0} \frac{d(e^x - e^{-x})}{2\frac{dx}{dx}}$

$= \lim\limits_{x \to 0} \frac{e^x + e^{-x}}{2} = \frac{e^0 + e^{-0}}{2} = 1$

Ex:-

$$\lim_{x\to 0} \frac{e^{x} + e^{-x} - 2x}{x - \sin x} \quad \left[\frac{0}{0}\right]$$ By L-Hospital Rule

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(e^{x} - e^{-x} - 2x)}{\frac{d}{dx}(x - \sin x)}$$

$$= \lim_{x\to 0} \frac{e^{x} + e^{-x} - 2}{1 - \cos x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{e^{x} - e^{-x}}{\sin x} \quad \left[\frac{0}{0}\right]$$ By L-Hospital Rule.

$$= \lim_{x\to 0} \frac{e^{x} + e^{-x}}{\cos x}$$

$$= \frac{e^{0} + e^{0}}{\cos 0} = \frac{2}{1} = 2.$$

Ex:→ Evaluate

$$\lim_{x\to 0} \frac{\cosh x - \cos x}{x \sin x} \quad \left[\text{Form } \frac{0}{0}\right]$$

using L-Hospital Rule.

$$\lim_{x\to 0} \frac{\frac{d}{dx}(\cosh x - \cos x)}{\frac{d}{dx}(x \sin x)}$$

$$= \lim_{x\to 0} \frac{\sinh x + \sin x}{\sin x + x\cos x} \quad \left[\frac{0}{0}\right]$$

Again Applying L-Hospital Rule.

$$= \lim_{x\to 0} \frac{\cosh x + \cos x}{\cos x + \cos x - x\sin x} = \frac{1+1}{2-0} = \frac{2}{2} = 1$$

Ex:->

$$\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2} \quad \left[\frac{0}{0}\right]$$

using L-Hopital Rule

$$\lim_{x\to 0} \frac{\frac{d}{dx}\left[xe^x - \log(1+x)\right]}{\frac{d}{dx}(x^2)}$$

$$= \lim_{x\to 0} \frac{e^x + xe^x - \frac{1}{1+x}}{2x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}\left[e^x + xe^x - \frac{1}{1+x}\right]}{2\frac{d}{dx}x}$$

$$= \lim_{x\to 0} \frac{e^x + e^x + xe^x + \frac{1}{(1+x)^2}}{2}$$

$$= \frac{e^0 + e^0 + 0 + \frac{1}{(1+0)^2}}{2} = \frac{1+1+1}{2} = \frac{3}{2}$$

$$\lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x} \quad \left[\frac{0}{0}\right]$$

Applying L-Hospital Rule.

$$\lim_{x\to 0} \frac{e^x - e^{\sin x}.\cos x}{1 - \cos x} \quad \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{e^x - \left[-\sin x\, e^{\sin x} + \cos x\, e^{\sin x}.\cos x\right]}{0 + \sin x}$$

$$= \lim_{x\to 0} \frac{e^x + e^{\sin x}\sin x - \cos x\, e^{\sin x}\cos x}{\cos x}$$

$$= \frac{1+1-1}{1} = 1$$